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Foundations of Query Languages
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2. Exercise Set: FOL and its Relationship to Databases

Exercise 1 (Modelling in FOL)

Consider the following database scheme $\{D, P, S, I\}$, where D, P, S are unary relational symbols and I is a 2-ary relational symbol. Let the database (D^A, P^A, S^A, I^A) be given. We interpret D^A as the set of exactly all points and straight lines. $P^A(a)$ means that " a is a point". $S^A(a)$ means that " a is a straight line". $I^A(a, b)$ means that " a is a point, b is a straight line and a lies on b ". Write down in first-order logic:

- a) For two points there is exactly one straight line on which they lie.
- b) On every straight line there lie at least two points.
- c) The intersection of two straight lines is exactly one point.

Exercise 2 (Query Reformulation)

Let

$$\varphi := \exists z(R(x, z) \wedge \exists w S(w, x, y)) \wedge \neg x = y$$

be given. Rewrite the query $\{ \langle x, y \rangle \mid \varphi \}$ to an equivalent formula in relational algebra. Do you have all information that you need in order to do so?

Hint: In order to do this exercise you may make further assumptions on the schema.

Exercise 3 (Query Satisfiability)

Is there a finite database such that the relational calculus query associated with φ yields a non-empty result? If so, give an example for such a structure (together with an underlying domain). If not, then prove your claim.

- $\varphi := \forall y(P(y, c) \Leftrightarrow P(y, y)) \wedge \neg P(x, x)$
- $\varphi := \forall y(P(y, c) \Leftrightarrow P(y, y)) \wedge \neg P(x, x) \wedge P(x, c)$

Assume that c is a constant.

Exercise 4 (Undecidability)

Let φ, ψ be first-order sentences as input. Conclude from Trakhtenbrot's Theorem that the following problems are undecidable:

- a) equivalence of φ and ψ ,
- b) containment of φ in ψ , and
- c) domain independence for input φ .

Due by: May 5, 2010 before the tutorial starts.

Literature: S. Abiteboul, R. Hull, V. Vianu: *Foundations of Databases*, Addison-Wesley, 1995. ISBN 0-201-53771-0. Download available at <http://www.inf.unibz.it/~nutt/FDBs0809>.